Problem Set 8 - Solution - LV 141.A55 QISS

1. Cooper-pair box

(a)
```
def CPB(EJ,Ec,ng,nmax):
    n=2*nmax+1
    Ham = 4*Ec*np.diag((np.arange(-nmax,nmax+1)-ng)**2)
    Ham += -0.5*EJ*(np.diag(np.ones(n-1),-1)+np.diag(np.ones(n-1),1))
    return Ham
```

(b)
```
import numpy as np
import matplotlib.pyplot as mpl

ng = np.linspace(0, 1, 201)
En = np.array([np.real(np.sort(np.linalg.eigvals(CPB(4, 1, n, 5)))[0:4]) for n in ng])

mpl.plot(ng, En)
mpl.xlabel('ng')
mpl.ylabel('Energy (E_c)')
mpl.grid(True)
mpl.savefig('fig1.pdf')
mpl.show()
```

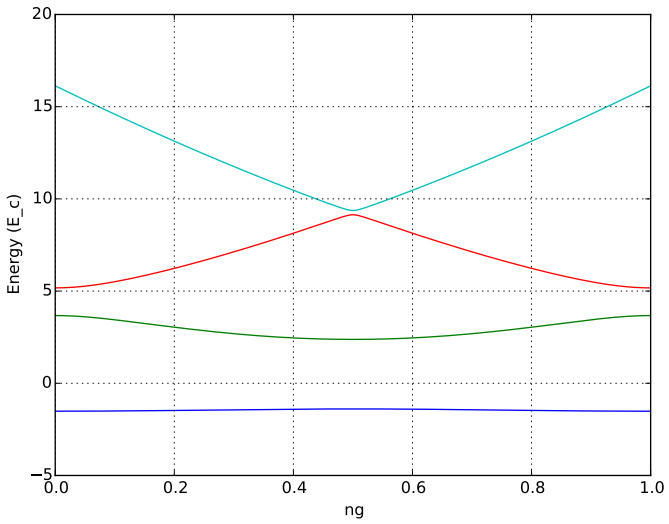

Figure 1: Energy Levels of a Cooper-pair box for $E_J = 4E_c$.

(c) Here we compare the solution against the two-level approximation result

$$\Delta E = \sqrt{16E_c^2(1-2n_g)^2 + E_J^2}$$

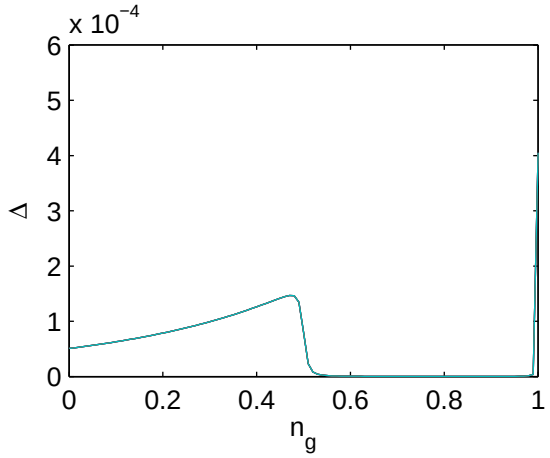

Figure 2: Accuracy of the Hamiltonian truncation. Comparing the 4th energy level solution taking into account charge state±3 or ±6. $\Delta = \frac{E_{4,n_{max}=3} - E_{4,n_{max}=6}}{E_{4,n_{max}=6}}$

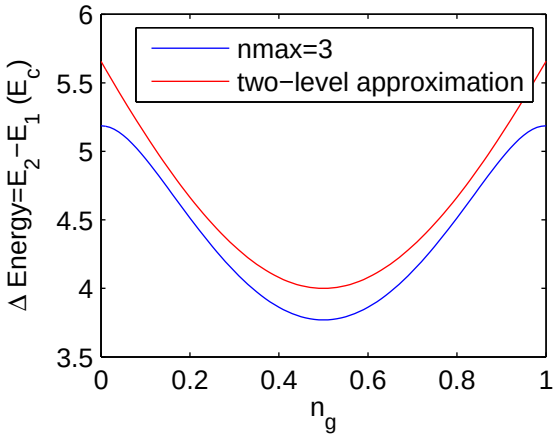

Figure 3: